JHEP HYPER VERSION

Received: *October 0, 2001*, Accepted: *October 29, 2001*

The high energy limit of QCD at two loops

Vittorio Del Duca

Istituto Nazionale di Fisica Nucleare, Sez. di Torino
via P. Giuria, 1 - 10125 Torino, Italy
E-mail: delduca@to.infn.it

E.W. Nigel Glover

Institute for Particle Physics Phenomenology, University of Durham
Durham, DH1 3LE, U.K.
E-mail: E.W.N.Glover@durham.ac.uk

Abstract: By taking the high-energy limit of the two-loop amplitudes for parton-parton scattering, we have tested the validity of the loop expansion of the high-energy amplitude, arising from a reggeized gluon passed in the crossed channel. As expected, we have found that it holds at LL and NLL accuracy, and hence we have independently re-evaluated the two-loop Regge trajectory, finding full agreement with the previous results by Fadin and collaborators. We have found, though, that the universality implied by the exchange of a single reggeized gluon in the crossed channel is violated at the next-to-next-to-leading logarithmic level.

Keywords: QCD, NLO Computations, Hadronic Colliders.

JHEP10(2001)035

Contents

1. Introduction

In the limit of squared center-of-mass energy much greater than the momentum transfer, $s \gg |t|$, any QCD scattering process is dominated by gluon exchange in the crossed channel.[1] Building upon this fact, the BFKL theory models strong-interaction processes with two large and disparate scales, by resumming the radiative corrections to parton-parton scattering. This is achieved to leading logarithmic (LL) accuracy, in $\ln(s/|t|)$, through the BFKL equation [1, 2, 3], i.e. a two-dimensional integral equation which describes the evolution of the t-channel gluon propagator in transverse momentum space and moment space. The integral equation is obtained by computing the one-loop LL corrections to the gluon exchange in the t channel. They are formed by a real correction: the emission of a gluon along the ladder [4], and a virtual correction: the so-called one-loop *Regge trajectory* (see eq. (2.2)). The BFKL equation is then obtained by iterating recursively these one-loop corrections to all orders in α_s, to LL accuracy. The calculation of the building blocks necessary to evaluate the next-to-leading logarithmic (NLL) corrections to the BFKL equation spanned over a decade. They are the emission of two gluons or two quarks along the ladder [5]–[8], the one-loop corrections to the emission of a gluon along the ladder [9]–[13], and the two-loop Regge trajectory [14]–[17]. The NLL corrections to the BFKL equation itself have been computed in refs. [18, 19, 20].

[1] For the sake of notational simplicity, we omit the carets on the partonic kinematic variables.

In this paper we explicitly take the high-energy limit of the two-loop amplitudes for parton-parton scattering [21]–[24]. This allows us to re-evaluate, in a fully independent way, the two-loop Regge trajectory, and to explore the possibility of extending the BFKL resummation beyond NLL accuracy.

2. Virtual corrections in the high-energy limit

In the high-energy limit $s \gg |t|$, any scattering process is dominated by gluon exchange in the t channel. In this context, the simplest process is parton-parton scattering, for which gluon exchange in the t channel occurs already at leading order (LO) in perturbative QCD. Thus we shall use it as a paradigm. The amplitude for parton-parton scattering $i_a\, j_b \to i_{a'}\, j_{b'}$, with i, j either a quark or a gluon, may be written as [1],

$$\mathcal{M}^{(0)aa'bb'}_{ij\to ij} = 2s \left[g_S\, (T^c_r)_{aa'}\, C^{i(0)}(p_a, p_{a'})\right] \frac{1}{t} \left[g_S\, (T^c_r)_{bb'}\, C^{j(0)}(p_b, p_{b'})\right], \tag{2.1}$$

where a, a', b, b' represent the colours of the scattering partons, and r represents either the fundamental (F) or the adjoint (G) representations of SU(N), with $(T^c_G)_{ab} = i f^{acb}$ and $\mathrm{tr}(T^c_F T^d_F) = \delta^{cd}/2$. The coefficient functions $C^{i(0)}$, which yield the LO impact factors, are given in ref. [1] in terms of their spin structure and in ref. [25, 26] at fixed helicities of the external partons. The square of the amplitude (2.1), integrated over the phase space, yields the parton-parton production rate to LO, $\mathcal{O}(\alpha_S^2)$, in the high-energy limit.

The virtual radiative corrections to eq. (2.1) in LL approximation are obtained, to all orders in α_S, by replacing [1]

$$\frac{1}{t} \to \frac{1}{t} \left(\frac{s}{-t}\right)^{\alpha(t)}, \tag{2.2}$$

in eq. (2.1), where $\alpha(t)$ is related to the one-loop transverse-momentum integration. In dimensional regularization in $d = 4 - 2\epsilon$ dimensions, it can be written as

$$\alpha(t) = g_S^2\, C_A\, \frac{2}{\epsilon} \left(\frac{\mu^2}{-t}\right)^{\epsilon} c_\Gamma\,, \tag{2.3}$$

with $C_A = N$, and

$$c_\Gamma = \frac{1}{(4\pi)^{2-\epsilon}} \frac{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\,. \tag{2.4}$$

The fact that higher order corrections to gluon exchange in the t channel can be accounted for by dressing the gluon propagator with the exponential of eq. (2.2) is what is called the reggeization, or the Regge trajectory, of the gluon, and, as said in the Introduction, lies at the core of the BFKL program.

JHEP10(2001)035

In order to go beyond the LL approximation, we need a prescription that allows us to disentangle the virtual corrections to the coefficient functions in eq. (2.1) from the ones that reggeize the gluon (2.2) within a loop amplitude. Such a prescription is supplied by the general form of the high-energy amplitude for parton-parton scattering, arising from a single reggeized gluon passed in the crossed channel. For quark-quark scattering, it can be written as [27],

$$\begin{aligned}\mathcal{M}^{aa'bb'}_{qq\to qq} = {} & s[g_S\,(T^c_F)_{aa'}\,C^q(p_a,p_{a'})]\frac{1}{t}\left[\left(\frac{-s}{-t}\right)^{\alpha(t)}+\left(\frac{s}{-t}\right)^{\alpha(t)}\right][g_S\,(T^c_F)_{bb'}\,C^q(p_b,p_{b'})]+\\ & +\frac{N^2-4}{N^2}s\,[g_S\,(T^c_F)_{aa'}\,C^q(p_a,p_{a'})]\,\frac{1}{t}\left[\left(\frac{-s}{-t}\right)^{\alpha(t)}-\left(\frac{s}{-t}\right)^{\alpha(t)}\right]\times\\ & \times[g_S\,(T^c_F)_{bb'}\,C^q(p_b,p_{b'})]+\cdots\,, \end{aligned} \tag{2.5}$$

and for quark-gluon and gluon-gluon scattering, it is [9],

$$\begin{aligned}\mathcal{M}^{aa'bb'}_{ig\to ig} = {} & s\,\left[g_S\,(T^c_r)_{aa'}\,C^i(p_a,p_{a'})\right]\frac{1}{t}\left[\left(\frac{-s}{-t}\right)^{\alpha(t)}+\left(\frac{s}{-t}\right)^{\alpha(t)}\right]\times\\ & \times[g_S\,(T^c_G)_{bb'}\,C^g(p_b,p_{b'})]+\cdots\,,\end{aligned} \tag{2.6}$$

with $r = F(G)$ for $i = q(g)$. eqs. (2.5) and (2.6) are symbolically represented in figure 1. The first (second) line of eqs. (2.5) and (2.6) corresponds to the exchange of a reggeized gluon of negative (positive) signature, belonging to the antisymmetric (symmetric) representation of SU(N). The dots at the end of eqs. (2.5) and (2.6) account for the (yet unknown) exchange of three or more reggeized gluons as well as other colour structures that vanish when contracted with the tree amplitude.

In this paper, we are only interested in terms that survive when projected by tree-level. In multiplying eq. (2.6) by the tree amplitude, the symmetric part of the reggeized gluon does not contribute, since the colour factor of the tree quark-gluon and gluon-gluon amplitudes, contains at least one structure constant, $f^{bdb'}$, which acts as an s-channel projector [28], thus singling out the antisymmetric gluon exchange.

Therefore, the positive signature contribution is only present for quark-quark scattering (2.5).

The gluon Regge trajectory has the perturbative expansion,

$$\alpha(t) = \tilde{g}_S^2\alpha^{(1)}(t) + \tilde{g}_S^4\alpha^{(2)}(t) + \mathcal{O}(\tilde{g}_S^6)\,, \tag{2.7}$$

with $\alpha^{(1)}(t)$ given in eq. (2.3), while the impact factor can be written as

$$C^i = C^{i(0)}(1 + \tilde{g}_S^2 C^{i(1)} + \tilde{g}_S^4 C^{i(2)}) + \mathcal{O}(\tilde{g}_S^6)\,. \tag{2.8}$$

In eqs. (2.7) and (2.8), we found convenient to rescale the coupling,

$$\tilde{g}_S^2 = g_S^2 c_\Gamma\left(\frac{\mu^2}{-t}\right)^\epsilon\,. \tag{2.9}$$

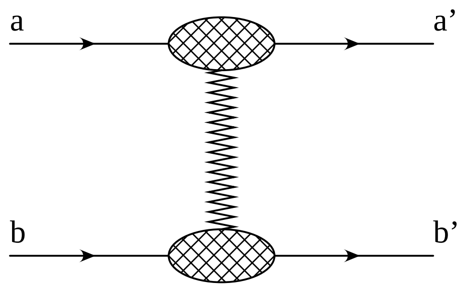

Figure 1: The symbolic representation of the factorised form for the high energy limit of the parton-parton scattering amplitude. The blobs represent the coefficient functions $C^i(p_a, p_{a'})$ (for $i = g,\ q$) while the zigzag line describes the reggeized gluon exchange.

Then we can write the projection of the amplitudes (2.5) and (2.6) on the tree amplitude as an expansion proportional to the tree amplitude squared,

$$\mathcal{M}^{aa'bb'}_{ij\to ij}\mathcal{M}^{(0)aa'bb'}_{ij\to ij} = |\mathcal{M}^{(0)aa'bb'}_{ij\to ij}|^2 \left(1 + \tilde{g}_S^2\ M^{(1)aa'bb'}_{ij\to ij} + \tilde{g}_S^4 M^{(2)aa'bb'}_{ij\to ij} + \mathcal{O}(\tilde{g}_S^6)\right), \quad (2.10)$$

with $i, j = g, q$. The one-loop coefficient of eq. (2.10) is,

$$M^{(1)aa'bb'}_{ij\to ij} = \alpha^{(1)}(t) \ln\left(\frac{s}{-t}\right) +\ C^{i(1)} + C^{j(1)} - i\frac{\pi}{2}\left(1 + \kappa\frac{N^2-4}{N^2}\right)\alpha^{(1)}(t)\,, \quad (2.11)$$

where $\kappa = 1$ for quark-quark scattering, and $\kappa = 0$ in the other cases. In eq. (2.11) we used the usual prescription $\ln(-s) = \ln(s) - i\pi$, for $s > 0$. Schematically, this is illustrated in figure 2. The LL reggeization term, $\alpha^{(1)}(t) = 2C_A/\epsilon$, is independent of the type of parton undergoing the high-energy scattering process (it is *universal*). It is also independent of the infrared (IR) regularisation scheme which is used. Conversely, the one-loop coefficient functions, $C^{i(1)}$, are process and IR-scheme dependent. The $C^{i(1)}$'s were computed in conventional dimensional regularization (CDR)/ 't-Hooft-Veltman (HV) schemes[2] in ref. [9, 13] and [30]–[33], and in the dimensional reduction scheme in ref. [13, 33]. According to eqs. (2.5) and (2.6), the coefficient functions C^i are real, the imaginary part of the amplitude being yielded by the trajectory. In addition to octet exchange, the explicit calculation of the imaginary part of the one-loop amplitude in the high-energy limit [33] yields other colour structures. This fact does not invalidate the NLL program, which is based on the validity of the antisymmetric part of eqs. (2.5) and (2.6), since the imaginary part of the one-loop amplitude does not contribute to the NLL corrections to the BFKL resummation.

[2]At the amplitude level, the difference between the CDR and the HV schemes, which resides in the number of helicities of the external gluons, is $\mathcal{O}(\epsilon)$ [29]. This difference only affects the pole structure at the squared amplitude level.

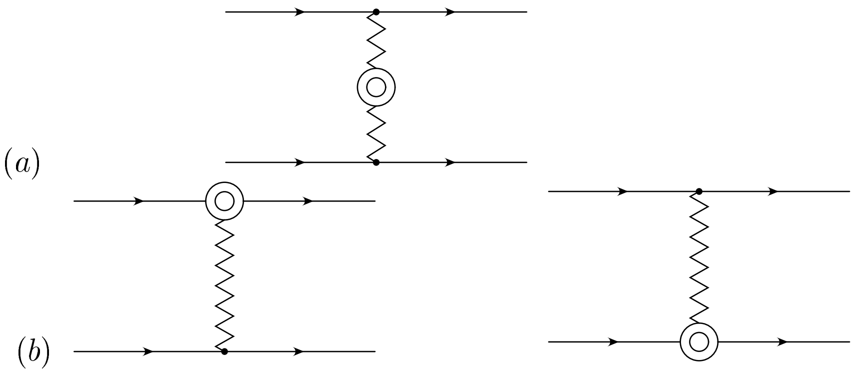

Figure 2: Schematic one-loop expansion of the factorised form for the high energy limit of the parton-parton scattering amplitude. The pairs of concentric circles represent the one-loop corrections to the impact factor and regge trajectory and the individual diagrams represent terms that contribute at (a) leading and (b) next-to-leading logarithmic order.

The two-loop coefficient of eq. (2.10) is,

$$
\begin{aligned}
M^{(2)aa'bb'}_{ij\to ij} &= \frac{1}{2}\left(\alpha^{(1)}(t)\right)^2 \ln^2\left(\frac{s}{-t}\right) + \\
&+ \left[\alpha^{(2)}(t) + \left(C^{i(1)} + C^{j(1)}\right)\alpha^{(1)}(t) - i\frac{\pi}{2}\left(1+\kappa\frac{N^2-4}{N^2}\right)\left(\alpha^{(1)}(t)\right)^2\right] \times \\
&\times \ln\left(\frac{s}{-t}\right) + \\
&+ \left[C^{i(2)} + C^{j(2)} + C^{i(1)}\,C^{j(1)} - \frac{\pi^2}{4}\left(1+\kappa\frac{N^2-4}{N^2}\right)\left(\alpha^{(1)}(t)\right)^2\right] - \\
&- i\frac{\pi}{2}\left(1+\kappa\frac{N^2-4}{N^2}\right)\left[\alpha^{(2)}(t) + \left(C^{i(1)} + C^{j(1)}\right)\alpha^{(1)}(t)\right].
\end{aligned} \tag{2.12}
$$

Schematically, this is illustrated in figure 3.

The first line of eq. (2.12) is just the exponentiation of the one-loop trajectory (figure 3(a)). If the single-log term is known, the second line of eq. (2.12) allows to determine $\alpha^{(2)}(t)$, the two-loop Regge trajectory (The first diagram in figure 3(b)). The third and fourth lines are respectively the real (figure 3(c)) and the imaginary parts of the constant term. Note that the gluon of positive signature does not contribute to the BFKL resummation at LL and NLL accuracy. The two-loop coefficient functions $C^{i(2)}$ could in principle be used to construct the next-to-next-to-leading order (NNLO) impact factors, if the BFKL resummation held to next-to-next-to-leading-log (NNLL) accuracy.

3. The two-loop amplitude in the high-energy limit

As indicated in the Introduction, we wish to make an independent check of the two-loop trajectory of the reggeized gluon that is exchanged in parton-parton scattering

JHEP10(2001)035

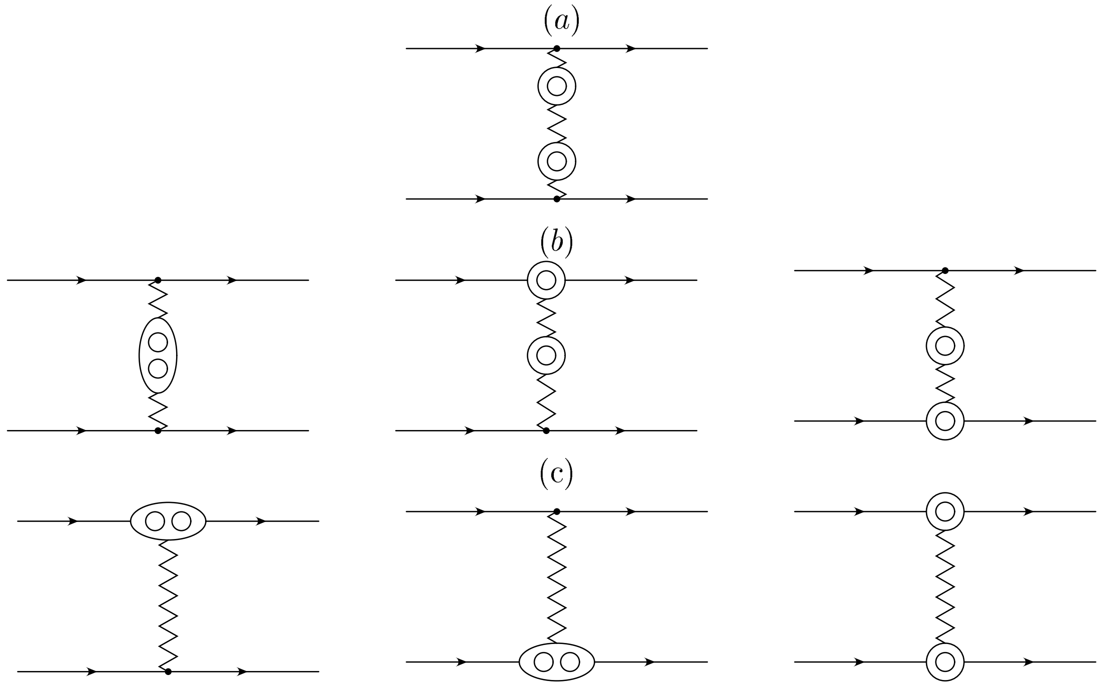

Figure 3: Schematic two-loop expansion of the factorised form for the high energy limit of the parton-parton scattering amplitude. The combinations of ovals and circles represent the one-loop and two-loop corrections to the impact factor and regge trajectory and the individual diagrams represent terms that contribute at (a) leading, (b) next-to-leading and (c) next-to-next-to-leading logarithmic order.

processes in the high energy limit. We wish to do this directly by taking the high energy limit of the two-loop parton-parton scattering amplitudes. The interference of the tree- and two-loop amplitudes for each of the parton-parton scattering processes have been explicity computed in refs. [21]–[24] using conventional dimensional regularization (CDR) and renormalised in the $\overline{\mathrm{MS}}$ scheme. In these papers the divergent contribution is written in terms of the infrared singularity operators $\mathbf{I}^{(1)}$, $\mathbf{I}^{(2)}$ and $\mathbf{H}^{(2)}$ proposed by Catani [34] and the tree- and one-loop amplitudes. The finite remainder is given in terms of logarithms and polylogarithms with arguments $-u/s$, $-t/s$ and u/t. This latter argument can be flipped using standard polylogarithm identities so that the $t \to 0$ limit for the $gg \to gg$, $qg \to qg$ and $qq' \to qq'$ processes can be straightforwardly taken. After expansion in ϵ, the leading power in s/t of the interference between the (unrenormalised) n-loop and the tree amplitudes for $ij \to ij$ has the form

$$\mathrm{Re}\ \left(\mathcal{M}^{(0)*}\mathcal{M}^{(n)}\right)_{ij\to ij} = |\mathcal{M}^{(0)}|^2_{ij\to ij}\ \tilde{g}_S^{2n} \sum_{m=0}^{n} B^{ij}_{nm} \ln^m\left(-\frac{s}{t}\right), \tag{3.1}$$

with $\tilde{g}_S^2$ given in eq. (2.9), and where $|\mathcal{M}^{(0)}|^2_{ij\to ij}$ is the high energy limit of the square of the tree-amplitude in CDR. For $n = 0$, $B^{ij}_{00} = 1$.

By explicit comparison of the leading singularity in t with the general expression given in eq. (3.1), we find the following relations

$$B_{11}^{ij} = \frac{2}{\epsilon} C_A \tag{3.2}$$

$$B_{10}^{qg} = \frac{1}{2}\left(B_{10}^{gg} + B_{10}^{qq}\right) \tag{3.3}$$

$$B_{22}^{ij} = \frac{1}{2}\left(B_{11}^{ij}\right)^2 \tag{3.4}$$

$$B_{21}^{ij} = B_{11}^{ij} B_{10}^{ij} + C_A \beta_0 \frac{2}{\epsilon^2} + C_A K \frac{2}{\epsilon} + C_A^2 \left(\frac{404}{27} - 2\zeta_3\right) + C_A N_F \left(-\frac{56}{27}\right), \tag{3.5}$$

where

$$\beta_0 = \frac{(11C_A - 2N_F)}{6}, \qquad K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} N_F \,. \tag{3.6}$$

We compare now these relations to eqs. (2.11) and (2.12). At this point, a caveat is in order: since we perform the comparison at the level of the interference with the tree amplitude, we shall miss any colour structure, which might appear in the two-loop amplitude in the high-energy limit, but that is projected out by the interference with the tree amplitude. Therefore a successful comparison between eqs. (2.11) and (2.12) and eqs. (3.2)–(3.5) is a necessary but not sufficient condition for the validity of eqs. (2.5) and (2.6).

Eq. (3.2) verifies the universality of the one-loop trajectory, and eq. (3.4) its exponentiation at the two-loop level (see the first line of eq. (2.12))

$$B_{11}^{ij} = \alpha^{(1)}(t)\,; \qquad B_{22}^{ij} = \frac{1}{2}\left(\alpha^{(1)}(t)\right)^2 \,. \tag{3.7}$$

The system formed by eq. (2.11) for gluon-gluon, quark-quark and quark-gluon scattering is overconstrained, namely we have three equations and only two unknowns, the one-loop coefficients $C^{g(1)}$ and $C^{q(1)}$. For instance, we can use the one-loop amplitudes for gluon-gluon and quark-quark scattering to determine $C^{g(1)}$ and $C^{q(1)}$, respectively. Then the constant term of the amplitude for quark-gluon scattering can be obtained without any further calculation. Conversely, the explicit calculation of quark-gluon scattering (see appendix A) tests eq. (3.3) and thus the validity of the high-energy expansion to one-loop accuracy,

$$C^{g(1)} = \frac{1}{2} B_{10}^{gg}\,; \quad C^{q(1)} = \frac{1}{2} B_{10}^{qq}\,; \quad B_{10}^{qg} = C^{g(1)} + C^{q(1)} = \frac{1}{2}\left(B_{10}^{gg} + B_{10}^{qq}\right). \tag{3.8}$$

Comparing eq. (3.5) to the single-log term of eq. (2.12) determines the value and verifies the universality of the two-loop trajectory,

$$B_{21}^{ij} = \alpha^{(2)}(t) + \alpha^{(1)}(t)(C^{i(1)} + C^{j(1)})\,. \tag{3.9}$$

The (unrenormalised) two-loop trajectory is

$$\begin{aligned}
\alpha^{(2)}(t) &= B_{21}^{ij} - B_{11}^{ij} B_{10}^{ij} \\
&= C_A \beta_0 \frac{2}{\epsilon^2} + C_A K \frac{2}{\epsilon} + C_A^2 \left(\frac{404}{27} - 2\zeta_3\right) + C_A N_F \left(-\frac{56}{27}\right), \qquad (3.10)
\end{aligned}$$

in agreement with the unrenormalised two-loop trajectory of ref. [14]–[17], and [35].

If the general form of the high-energy scattering amplitudes (2.5) and (2.6) holds to NNLO, and thus the expansion (2.12) is valid up to the constant terms, we can determine the two-loop coefficient functions $C^{i(2)}$ through B_{20}^{ij},

$$\begin{aligned}
B_{20}^{ij} &= C^{i(2)} + C^{j(2)} + C^{i(1)}\, C^{j(1)} - \frac{\pi^2}{4} \left(\alpha^{(1)}(t)\right)^2, \qquad i = q, g \;\; j = g \\
B_{20}^{qq} &= 2C^{q(2)} + \left(C^{q(1)}\right)^2 - \frac{\pi^2}{4} \left(1 + \frac{N^2 - 4}{N^2}\right) \left(\alpha^{(1)}(t)\right)^2. \qquad (3.11)
\end{aligned}$$

As for eq. (2.11), the system formed by eq. (3.11) for gluon-gluon, quark-quark and quark-gluon scattering has three equations and only two unknowns, the one-loop coefficients $C^{g(2)}$ and $C^{q(2)}$. We can use the two-loop amplitudes for gluon-gluon and quark-quark scattering to determine $C^{g(2)}$ and $C^{q(2)}$, respectively. Then the validity of the high-energy expansion to two-loop accuracy implies the relation,

$$B_{20}^{qg} - \frac{1}{4} B_{10}^{qq} B_{10}^{gg} - \frac{\pi^2}{8} \frac{N^2 - 4}{N^2} \left(\alpha^{(1)}(t)\right)^2 = \frac{1}{2} \left[B_{20}^{gg} - \frac{1}{4} \left(B_{10}^{gg}\right)^2 + B_{20}^{qq} - \frac{1}{4} \left(B_{10}^{qq}\right)^2 \right]. \qquad (3.12)$$

Using eq. (3.3), this can be recast as the difference between terms depending on the quark-gluon amplitude and terms that depend on the gluon-gluon and quark-quark amplitudes,

$$B_{20}^{qg} - \frac{1}{2} \left(B_{10}^{qg}\right)^2 - \frac{1}{2} \left[B_{20}^{gg} - \frac{1}{2} \left(B_{10}^{gg}\right)^2 + B_{20}^{qq} - \frac{1}{2} \left(B_{10}^{qq}\right)^2 \right] - \frac{\pi^2}{2\epsilon^2} \left(N^2 - 4\right) = 0. \qquad (3.13)$$

Through the explicit calculation of the B^{ij} coefficients (see appendix A) we found that eq. (3.13) holds for constant and ζ_3 terms but that it is violated by terms of $\mathcal{O}(\pi^2/\epsilon^2)$,

$$\begin{aligned}
B_{20}^{qg} - \frac{1}{2} \left(B_{10}^{qg}\right)^2 - \frac{1}{2} \left[B_{20}^{gg} - \frac{1}{2} \left(B_{10}^{gg}\right)^2 + B_{20}^{qq} - \frac{1}{2} \left(B_{10}^{qq}\right)^2 \right] - \frac{\pi^2}{2\epsilon^2} \left(N^2 - 4\right) &= \\
= \frac{3\pi^2}{\epsilon^2} \left(\frac{N^2 + 1}{N^2}\right) + \mathcal{O}(\epsilon). \qquad (3.14)
\end{aligned}$$

This violation is due to the exchange of three or more reggeized gluons which is unaccounted for in eqs. (2.5) and (2.6).

Analogously to eq. (3.1), we can write the leading power in s/t of the interference between the unrenormalised n-loop and the tree amplitudes as,

$$\mathrm{Im}\, \left(\mathcal{M}^{(0)*} \mathcal{M}^{(n)}\right)_{ij\to ij} = -\frac{\pi}{2} \left|\mathcal{M}^{(0)}\right|^2_{ij\to ij} \tilde{g}_S^{2n} \sum_{m=0}^{n-1} D_{nm}^{ij} \ln^m \left(-\frac{s}{t}\right), \qquad (3.15)$$

with $n \geq 1$.

JHEP10(2001)035

By explicit comparison of the leading singularity in t with the general expression given in eq. (3.15), and by using eqs. (2.3) and (3.9), we find,

$$D_{10}^{ij} = \frac{2C_A}{\epsilon} = \alpha^{(1)}(t) \tag{3.16}$$

$$D_{21}^{ij} = \left(D_{10}^{ij}\right)^2 = \left(\alpha^{(1)}(t)\right)^2 \tag{3.17}$$

$$D_{20}^{ij} = B_{21}^{ij} = \left[\alpha^{(2)}(t) + \alpha^{(1)}(t)(C^{i(1)} + C^{j(1)})\right] \tag{3.18}$$

for $i = q, g$ and $j = g$, and

$$D_{10}^{qq} = \frac{4(4C_F - C_A)}{\epsilon} \tag{3.19}$$

$$D_{21}^{qq} = \frac{8C_A(4C_F - C_A)}{\epsilon^2} \tag{3.20}$$

$$D_{20}^{qq} = \frac{2(4C_F - C_A)}{C_A}\left[\alpha^{(2)}(t) + 2\alpha^{(1)}(t)C^{q(1)}\right] \tag{3.21}$$

for quark-quark scattering. Eqs. (3.16)–(3.21) are in agreement with the imaginary parts of eqs. (2.11) and (2.12). However, the same caveat we put forward after eq. (3.6) is valid here, namely we cannot exclude that other colour structures appear which are killed by the projection on the tree amplitude in the high-energy limit.

4. Conclusions

By taking the high-energy limit of the two-loop amplitudes for parton-parton scattering [21]–[24], we have tested the validity of the general form of the high-energy amplitudes (2.5) and (2.6) for parton-parton scattering, arising from a reggeized gluon exchanged in the crossed channel. As expected, we have found that it holds at LL and NLL accuracy, and hence we have independently re-evaluated the two-loop Regge trajectory (3.10), finding full agreement with ref. [14]–[17]. We have found, though, that the universality implied by eqs. (2.5) and (2.6) is violated at the next-to-next-to-leading order level, eq. (3.14). The source of the discrepancy between eqs. (3.13) and (3.14) might reside in the yet unknown exchange of three reggeized gluons, which is unaccounted for in eqs. (2.5) and (2.6).

Finally, we stress that our comparisons are done at the level of the interference between loop and tree amplitudes. Other colour structures may be present at the amplitude level, which may be killed, though, by the projection on the tree amplitude in the high-energy limit. Thus a more stringent comparison at the amplitude level would be welcome.

Acknowledgments

We thank the Les Houches Centre for Physics and the organizers of the Les Houches Workshop *Physics at TeV Colliders* where this work was initiated and Ugo Aglietti, Jochen Bartels, Stefano Catani, Victor Fadin, Lev Lipatov, Alan Martin, Zoltan Trocsanyi and Alan White for helpful comments. VDD also thanks the IPPP for its kind hospitality during the latter stages of this work.

A. The coefficients of the two-loop amplitude in the high-energy limit

In this appendix we give a complete list of the real and imaginary coefficients B^{ij}_{nm} and D^{ij}_{nm} obtained by expanding the interference of tree and two-loop graphs in the high energy limit. All results are valid in conventional dimensional regularisation. The one-loop coefficients are expanded keeping terms through to $\mathcal{O}(\epsilon^2)$ while the two-loop coefficients are given up to $\mathcal{O}(\epsilon)$.

A.1 gluon-gluon scattering

For the interference of tree with one-loop for gluon-gluon scattering we find,

$$\begin{aligned}
B^{gg}_{11} &= \frac{2}{\epsilon}C_A \\
B^{gg}_{10} &= C_A\left(-\frac{4}{\epsilon^2}+\left(-\frac{67}{9}+\pi^2\right)+\left(-\frac{422}{27}+2\zeta_3\right)\epsilon+\left(-\frac{2626}{81}+\frac{\pi^4}{15}\right)\epsilon^2\right)+ \\
&\quad +N_F\left(\frac{10}{9}+\frac{74}{27}\epsilon+\frac{580}{81}\epsilon^2\right)+\beta_0\left(-\frac{2}{\epsilon}\right) \\
D^{gg}_{10} &= B^{gg}_{11}
\end{aligned} \tag{A.1}$$

while the two-loop coefficients are given by,

$$\begin{aligned}
B^{gg}_{22} &= \frac{1}{2}\left(B^{gg}_{11}\right)^2 \\
B^{gg}_{21} &= B^{gg}_{11}B^{gg}_{10}+C_A\beta_0\frac{2}{\epsilon^2}+C_AK\frac{2}{\epsilon}+C_A^2\left(\frac{404}{27}-2\zeta_3\right)+C_AN_F\left(-\frac{56}{27}\right) \\
B^{gg}_{20} &= \frac{1}{2}\left(B^{gg}_{10}\right)^2-C_A\beta_0\frac{2}{\epsilon^3}-\beta_0^2\frac{2}{\epsilon^2}+ \\
&\quad +C_A^2\left(\left(-\frac{67}{9}-\frac{38\pi^2}{3}\right)\frac{1}{\epsilon^2}+\left(-\frac{1276}{27}+2\zeta_3+\frac{44\pi^2}{9}\right)\frac{1}{\epsilon}+\right. \\
&\qquad\left.+\left(-\frac{12433}{81}+44\zeta_3+\frac{268\pi^2}{27}-\frac{19\pi^4}{45}\right)\right)+C_AC_F\left(\frac{24\pi^2}{\epsilon^2}\right)+ \\
&\quad +C_AN_F\left(\frac{10}{9\epsilon^2}+\left(\frac{109}{9}-\frac{8\pi^2}{9}\right)\frac{1}{\epsilon}+\left(\frac{3169}{81}-\frac{40\pi^2}{27}\right)\right)+ \\
&\quad +C_FN_F\left(\frac{1}{\epsilon}+\left(\frac{55}{6}-8\zeta_3\right)\right)++N_F^2\left(-\frac{20}{27\epsilon}-\frac{22}{9}\right) \\
D^{gg}_{21} &= \left(B^{gg}_{11}\right)^2 \\
D^{gg}_{20} &= B^{gg}_{21}\,.
\end{aligned} \tag{A.2}$$

A.2 Quark-gluon scattering

For the interference of tree with one-loop for quark-gluon scattering we find,

$$
\begin{aligned}
B_{11}^{qg} &= \frac{2}{\epsilon} C_A \\
B_{10}^{qg} &= C_A \left(-\frac{2}{\epsilon^2} + 1 + \pi^2 + \left(\frac{5}{3} + 2\zeta_3 \right) \epsilon + \left(\frac{25}{9} + \frac{\pi^4}{15} \right) \epsilon^2 \right) + \\
&\qquad + C_F \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 - 16\epsilon - 32\epsilon^2 \right) + N_F \left(\frac{1}{3}\epsilon + \frac{14}{9}\epsilon^2 \right) \\
D_{10}^{qg} &= B_{11}^{qg}
\end{aligned} \tag{A.3}
$$

while the two-loop coefficients are given by,

$$
\begin{aligned}
B_{22}^{qg} &= \frac{1}{2} \left(B_{11}^{qg} \right)^2 \\
B_{21}^{qg} &= B_{11}^{qg} B_{10}^{qg} + C_A \beta_0 \frac{2}{\epsilon^2} + C_A K \frac{2}{\epsilon} + C_A^2 \left(\frac{404}{27} - 2\zeta_3 \right) + C_A N_F \left(-\frac{56}{27} \right) \\
B_{20}^{qg} &= \frac{1}{2} \left(B_{10}^{qg} \right)^2 - C_A \beta_0 \frac{1}{\epsilon^3} - C_F \beta_0 \frac{1}{\epsilon^3} + \\
&\quad + C_A^2 \left(\left(-\frac{67}{18} - \frac{17\pi^2}{6} \right) \frac{1}{\epsilon^2} + \left(-\frac{94}{27} + \frac{77\pi^2}{18} + \zeta_3 \right) \frac{1}{\epsilon} + \right. \\
&\qquad \left. + \left(\frac{1289}{162} + \frac{469\pi^2}{54} + \frac{34\zeta_3}{3} - \frac{29\pi^4}{60} \right) \right) + \\
&\quad + C_A C_F \left(\left(-\frac{83}{9} + \frac{25\pi^2}{6} \right) \frac{1}{\epsilon^2} + \left(-\frac{4129}{108} - \frac{11\pi^2}{18} + 13\zeta_3 \right) \frac{1}{\epsilon} + \right. \\
&\qquad \left. + \left(-\frac{91765}{648} - \frac{110\pi^2}{27} + \frac{184\zeta_3}{3} + \frac{11\pi^4}{36} \right) \right) + \\
&\quad + C_F^2 \left(\left(-\frac{3}{4} + \pi^2 - 12\zeta_3 \right) \frac{1}{\epsilon} + \left(-\frac{1}{8} + \frac{29\pi^2}{6} - 30\zeta_3 - \frac{11\pi^4}{45} \right) \right) + \\
&\quad + C_A N_F \left(\frac{5}{9\epsilon^2} + \left(\frac{1}{27} - \frac{7\pi^2}{9} \right) \frac{1}{\epsilon} + \left(-\frac{178}{81} - \frac{35\pi^2}{27} - \frac{16\zeta_3}{3} \right) \right) + \\
&\quad + C_F N_F \left(\frac{14}{9\epsilon^2} + \left(\frac{353}{54} + \frac{\pi^2}{9} \right) \frac{1}{\epsilon} + \left(\frac{7541}{324} + \frac{14\pi^2}{27} - \frac{4\zeta_3}{3} \right) \right) + \\
&\quad + N_F^2 \left(-\frac{2}{9} \right) \\
D_{21}^{qg} &= \left(B_{11}^{qg} \right)^2 \\
D_{20}^{qg} &= B_{21}^{qg}
\end{aligned} \tag{A.4}
$$

A.3 Quark-quark scattering

For the interference of tree with one-loop for quark-quark scattering we find,

$$
B_{11}^{qq} = \frac{2}{\epsilon} C_A
$$

JHEP10(2001)035

$$
\begin{aligned}
B_{10}^{qq} &= C_A\left(\left(\frac{85}{9}+\pi^2\right)+\left(\frac{512}{27}+2\zeta_3\right)\epsilon+\left(\frac{\pi^4}{15}+\frac{3076}{81}\right)\epsilon^2\right)+\\
&\quad +C_F\left(-\frac{4}{\epsilon^2}-\frac{6}{\epsilon}-16-32\epsilon-64\epsilon^2\right)+\\
&\quad +N_F\left(-\frac{10}{9}-\frac{56\epsilon}{27}-\frac{328\epsilon^2}{81}\right)+\beta_0\left(\frac{2}{\epsilon}\right)\\
D_{10}^{qq} &= -\frac{4}{\epsilon}C_A+\frac{16}{\epsilon}C_F
\end{aligned}
\tag{A.5}
$$

while the two-loop coefficients are given by,

$$
\begin{aligned}
B_{22}^{qq} &= \frac{1}{2}\left(B_{11}^{qq}\right)^2\\
B_{21}^{qq} &= B_{11}^{qq}B_{10}^{qq}+C_A\beta_0\frac{2}{\epsilon^2}+C_AK\frac{2}{\epsilon}+C_A^2\left(\frac{404}{27}-2\zeta_3\right)+C_AN_F\left(-\frac{56}{27}\right)\\
B_{20}^{qq} &= \frac{1}{2}\left(B_{10}^{qq}\right)^2-C_F\beta_0\frac{2}{\epsilon^3}+\beta_0^2\frac{2}{\epsilon^2}+\\
&\quad +C_A^2\left(-\frac{2\pi^2}{\epsilon^2}+\left(\frac{1088}{27}+\frac{11\pi^2}{3}\right)\frac{1}{\epsilon}+\left(\frac{4574}{27}+\frac{67\pi^2}{9}-\frac{64\zeta_3}{3}-\frac{49\pi^4}{90}\right)\right)+\\
&\quad +C_AC_F\left(\left(-\frac{166}{9}+\frac{37\pi^2}{3}\right)\frac{1}{\epsilon^2}+\left(-\frac{4129}{54}-\frac{11\pi^2}{9}+26\zeta_3\right)\frac{1}{\epsilon}+\right.\\
&\qquad\quad \left.+\left(-\frac{91765}{324}-\frac{220\pi^2}{27}+\frac{368\zeta_3}{3}+\frac{11\pi^4}{18}\right)\right)+\\
&\quad +C_F^2\left(-\frac{24\pi^2}{\epsilon^2}+\left(-\frac{3}{2}+2\pi^2-24\zeta_3\right)\frac{1}{\epsilon}+\left(-\frac{1}{4}+\frac{29\pi^2}{3}-60\zeta_3-\frac{22\pi^4}{45}\right)\right)+\\
&\quad +C_AN_F\left(\left(-\frac{325}{27}-\frac{2\pi^2}{3}\right)\frac{1}{\epsilon}+\left(-\frac{1175}{27}-\frac{10\pi^2}{9}-\frac{32\zeta_3}{3}\right)\right)+\\
&\quad +C_FN_F\left(\frac{28}{9\epsilon^2}+\left(\frac{326}{27}+\frac{2\pi^2}{9}\right)\frac{1}{\epsilon}\right)+\left(\frac{3028}{81}+\frac{28\pi^2}{27}+\frac{16\zeta_3}{3}\right)+\\
&\quad +N_F^2\left(\frac{20}{27\epsilon}+2\right)\\
D_{21}^{qq} &= -\frac{8}{\epsilon^2}C_A^2+\frac{32}{\epsilon^2}C_AC_F\\
D_{20}^{qq} &= \frac{-2(C_A-4C_F)}{C_A}B_{21}^{qq}\,.
\end{aligned}
\tag{A.6}
$$

References

[1] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Multi-reggeon processes in the Yang-Mills theory*, *Sov. Phys. JETP* **44** (1976) 443.

[2] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *The pomeranchuk singularity in non-abelian gauge theories Sov. Phys. JETP* **45** (1977) 199

[3] I.I. Balitsky and L.N. Lipatov, *The pomeranchuk singularity in quantum chromodynamics*, *Sov. J. Nucl. Phys.* **28** (1978) 822.

[4] L.N. Lipatov, *Reggeization of the vector meson and the vacuum singularity in nonabelian gauge theories*, *Sov. J. Nucl. Phys.* **23** (1976) 338.

[5] V.S. Fadin and L.N. Lipatov, *High-energy production of gluons in a quasimultiregge kinematics*, *Sov. Phys. JETP Lett.* **49** (1989) 352.

[6] V. Del Duca, *Real next-to-leading corrections to the multigluon amplitudes in the helicity formalism*, *Phys. Rev.* **D 54** (1996) 989 [hep-ph/9601211].

[7] V.S. Fadin and L.N. Lipatov, *Next-to-leading corrections to the BFKL equation from the gluon and quark production*, *Nucl. Phys.* **B 477** (1996) 767 [hep-ph/9602287].

[8] V. Del Duca, *Quark-antiquark contribution to the multigluon amplitudes in the helicity formalism*, *Phys. Rev.* **D 54** (1996) 4474 [hep-ph/9604250].

[9] V.S. Fadin and L.N. Lipatov, *Radiative corrections to QCD scattering amplitudes in a multi-Regge kinematics*, *Nucl. Phys.* **B 406** (1993) 259.

[10] V. Fadin, R. Fiore and A. Quartarolo, *Quark contribution to the reggeon-reggeon-gluon vertex in QCD*, *Phys. Rev.* **D 50** (1994) 5893 [hep-th/9405127].

[11] V.S. Fadin, R. Fiore and M.I. Kotsky, *Gribov's theorem on soft emission and the reggeon-reggeon-gluon vertex at small transverse momentum*, *Phys. Lett.* **B 389** (1996) 737 [hep-ph/9608229].

[12] V. Del Duca and C.R. Schmidt, *Virtual next-to-leading corrections to the lipatov vertex*, *Phys. Rev.* **D 59** (1999) 074004 [hep-ph/9810215].

[13] Z. Bern, V. Del Duca and C.R. Schmidt, *The infrared behavior of one-loop gluon amplitudes at next-to-next-to-leading order*, *Phys. Lett.* **B 445** (1998) 168 [hep-ph/9810409].

[14] V.S. Fadin, M.I. Kotsky and R. Fiore, *Gluon reggeization in QCD in the next-to-leading order*, *Phys. Lett.* **B 359** (1995) 181.

[15] V.S. Fadin, R. Fiore and M.I. Kotsky, *Gluon Regge trajectory in the two-loop approximation*, *Phys. Lett.* **B 387** (1996) 593 [hep-ph/9605357].

[16] V. Fadin, R. Fiore and A. Quartarolo, *Reggeization of quark-quark scattering amplitude in QCD*, *Phys. Rev.* **D 53** (1996) 2729 [hep-ph/9506432].

[17] J. Blümlein, V. Ravindran and W.L. van Neerven, *On the gluon Regge trajectory in* $O(\alpha_s^2)$, *Phys. Rev.* **D 58** (1998) 091502 [hep-ph/9806357].

[18] V.S. Fadin and L.N. Lipatov, *BFKL pomeron in the next-to-leading approximation*, *Phys. Lett.* **B 429** (1998) 127 [hep-ph/9802290].

[19] G. Camici and M. Ciafaloni, *Irreducible part of the next-to-leading bfkl kernel*, *Phys. Lett.* **B 412** (1997) 396 [hep-ph/9707390].

[20] M. Ciafaloni and G. Camici, *Energy scale(s) and next-to-leading BFKL equation*, *Phys. Lett.* **B 430** (1998) 349 [hep-ph/9803389].

[21] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to massless quark gluon scattering*, *Nucl. Phys.* **B 605** (2001) 486 [hep-ph/0101304].

[22] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to $q\bar{q} \to q'\bar{q}'$*, *Nucl. Phys.* **B 601** (2001) 318 [hep-ph/0010212].

[23] C. Anastasiou, E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to $q\bar{q} \to q\bar{q}$*, *Nucl. Phys.* **B 601** (2001) 341 [hep-ph/0011094].

[24] E.W.N. Glover, C. Oleari and M.E. Tejeda-Yeomans, *Two-loop QCD corrections to gluon gluon scattering*, *Nucl. Phys.* **B 605** (2001) 467 [hep-ph/0102201].

[25] V. Del Duca, *Equivalence of the Parke-Taylor and the Fadin-Kuraev-Lipatov amplitudes in the high-energy limit*, *Phys. Rev.* **D 52** (1995) 1527 [hep-ph/9503340].

[26] V. Del Duca, *Next-to-leading corrections to the BFKL equation*, hep-ph/9605404.

[27] L. N. Lipatov, *Pomeron in quantum chromodynamics*, in *Perturbative quantum chromodynamics*, 411-489, Mueller ed.

[28] J. Bartels, *Unitarity corrections to the lipatov pomeron and the four gluon operator in deep inelastic scattering in QCD*, *Z. Physik* **C 60** (1993) 471.

[29] Z. Kunszt, A. Signer and Z. Trócsányi, *One loop helicity amplitudes for all $2 \to 2$ processes in QCD and $N = 1$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 411** (1994) 397 [hep-ph/9305239].

[30] V. Fadin and R. Fiore, *Quark contribution to the gluon-gluon-reggeon vertex in QCD*, *Phys. Lett.* **B 294** (1992) 286.

[31] L.N. Lipatov, *Small-x physics in perturbative QCD*, *Phys. Rep.* **286** (1997) 131 [hep-ph/9610276].

[32] V. Fadin, R. Fiore and A. Quartarolo, *Radiative corrections to quark-quark reggeon vertex in QCD*, *Phys. Rev.* **D 50** (1994) 2265 [hep-ph/9310252].

[33] V. Del Duca and C.R. Schmidt, *Virtual next-to-leading corrections to the impact factors in the high-energy limit*, *Phys. Rev.* **D 57** (1998) 4069 [hep-ph/9711309].

[34] S. Catani, *The singular behaviour of QCD amplitudes at two-loop order*, *Phys. Lett.* **B 427** (1998) 161 [hep-ph/9802439].

[35] V.S. Fadin, *BFKL news*, hep-ph/9807528.

[36] S. Catani, M.H. Seymour and Z. Trócsányi, *Regularization scheme independence and unitarity in QCD cross sections*, *Phys. Rev.* **D 55** (1997) 6819 [hep-ph/9610553].

JHEP10(2001)035